

One-Loop Supergravity Corrections to the Black Hole Entropy and Residual Supersymmetry

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Abstract

We study the one-loop corrections to the effective on-shell action of N=2 supergravity in the background of the Reissner-Nordstrom black hole. In the extreme case the contributions from graviton, gravitino and photon to the one-loop corrections to the entropy are shown to cancel. This gives the first explicit example of the supersymmetric non-renormalization theorem for the on-shell action (entropy) for BPS configurations which admit Killing spinors. We display the residual supersymmetry of the perturbations of a general supersymmetric theory in a bosonic BPS background.

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It was shown by Gibbons and Hawking [1] for the Reissner-Nordstrom black holes and in [2,3] for dilaton black holes that the Bekenstein-Hawking entropy of black holes can be found by evaluating the Euclidean on-shell action of the theory in the semiclassical approximation. The first quantum corrections to the entropy can be found by evaluating the one-loop partition function in the black hole background.

The arguments in favor of a supersymmetric non-renormalization theorem for the on-shell action (entropy) for configurations which admit Killing spinors were presented in [4,2,5]. The proof is based on Berezin rules of integration in superspace and the fact that Killing spinors can be related to isometries in fermionic directions in superspace. The simplest configuration constitutes the extreme Reissner-Nordstrom (RN) black hole of pure $N = 2$ supergravity [6]. The existence of Killing spinors for this solution was established in [7]. However, there was no explicit one-loop calculation available to support or disprove the theorem of non-renormalization of the entropy, not even in the RN case.

The situation with the one-loop corrections in the $N = 2$ theory was obscured by the existence of the so-called conformal anomalies [18]. The trace anomaly of the one-loop on-shell supergravity in a gravitational background is given by the following expression:

$$T = g^{\mu\nu} \langle T_{\mu\nu} \rangle = g^{\mu\nu} \frac{\partial}{\partial g^{\mu\nu}} \left\{ \frac{(\det D_{3/2})^2}{\det D_2 \det D_1} \right\} = \frac{A}{32\pi^2} {}^*R_{\mu\nu\lambda\delta} {}^*R^{\mu\nu\lambda\delta} . \quad (1)$$

Here D_s denotes the contribution from positive and negative helicity states of spin $s = 2$, two spin $s = 3/2$, and spin $s = 1$ fields of the full $N = 2$ supergravity multiplet. The coefficient A is known for all fields interacting with gravity.

The one-loop counterterm is proportional to the Euler number of the manifold, $S_{\text{one-loop}} = \frac{1}{\epsilon} A \chi$. The fields of $N = 2$ supergravity include a graviton, 2 gravitini and a vector field. The anomaly coefficient $A = \frac{11}{12}$ of pure $N = 2$ supergravity does not vanish. This means that in a *purely gravitational* background the contributions from spin 2, two spin 3/2, and spin 1 fields of the full $N = 2$ supergravity multiplet do not cancel in the first loop. Although those backgrounds do not admit Killing spinors, hence are not relevant for the proving or disproving the non-renormalization theorem, there was not much incentive

in calculating one-loop corrections in pure $N = 2$ supergravity in the background of the extreme Reissner-Nordstrom black hole. On the other hand, in the $N = 4$ theory where the conformal anomalies are absent, the calculations may be much more complicated and they have not been done either. In this paper, we will see that the one-loop contributions within the supergravity multiplet cancel in a background admitting a Killing spinor.

Meanwhile from a completely different perspective, a study of quasinormal modes of various black holes was developed (see [11]). Quasinormal modes of black holes provide an opportunity to identify black holes when large-scale laser-interferometric detectors for gravitational waves will be available. The formalism was applied in the study of black holes with general dilaton coupling constant a in [9]. In most cases only numerical calculations are possible. By applying those methods to the RN black hole H. Onozawa, T. Mishima, T. Okamura, and H. Ishihara [10] discovered a curious fact: the resonant frequencies of $s = 2$ waves with multipole index j_s coincide with those of $s = 1$ waves with multipole index $j_s - 1$ in the extremal limit. This would be very difficult to establish by looking directly into the standard form of the wave equations in generic, non-extreme RN as given in the book of Chandrasekhar [11] in ch. 5, eq. (270). The potential in the wave equation for $s = 1, 2$ (photons and gravitons) is defined there in terms of two different functions f_1 and f_2 and two different parameters q_1 and q_2 ¹

$$V_s^\pm = \pm q_s \frac{df_s}{dr_*} + q_s^2 f_s^2 + [A_s(A_s - 2)]f_s \quad s = 1, 2. \quad (2)$$

Here

$$f_s = \frac{\Delta}{r^3[(A_s - 2)r + q_s]} \quad (3)$$

and q_s, A_s, Δ are defined below. The graviton and photon perturbations are encoded in functions Z_1 and Z_2 [11] which satisfy the radial equations²

¹The notation for q_1 and q_2 is reversed in [11].

²The radial equations here describe the axial perturbations with $V_s^- \equiv V_s$. The polar perturba-

$$\mathcal{D}_s Z_s(r) \equiv \left[\frac{d^2}{dr_*^2} + \omega^2 - V_s(r) \right] Z_s(r) = 0, \quad (4)$$

where

$$\frac{dr}{dr_*} = \frac{\Delta}{r^2}, \quad \Delta = r^2 - 2Mr + Q^2, \quad (5)$$

$$V_s = \frac{\Delta}{r^5} \left[A_s r - q_s + \frac{4Q^2}{r} \right], \quad (6)$$

$$q_1 = 3M - \sqrt{9M^2 + 4Q^2(A_2 - 2)}, \quad (7)$$

$$q_2 = 3M + \sqrt{9M^2 + 4Q^2(A_1 - 2)}, \quad (8)$$

$$A_s = j_s(j_s + 1), \quad (9)$$

$$j_s = l + s, \quad (l = 0, 1, 2, \dots). \quad (10)$$

Here M and Q are the mass and charge of the black hole, and j_s is an angular multipole index of the perturbation. In the Schwarzschild case $Q = 0$, $s = 1$ and $s = 2$ correspond to the electromagnetic and the gravitational perturbation, respectively. For non-vanishing charge however, Z_1 and Z_2 are linear combinations of graviphoton and graviton perturbations of the same multipole index j_s . Schematically, this can be understood from the quadratic action

$$S^{(2)} = \frac{1}{2} \left(\Delta g \frac{\delta^2 S}{\delta g \delta g} \Delta g + 2 \Delta g \frac{\delta^2 S}{\delta g \delta A} \Delta A + \Delta A \frac{\delta^2 S}{\delta A \delta A} \Delta A + \Delta \Psi \frac{\delta^2 S}{\delta \Psi \delta \Psi} \Delta \Psi \right), \quad (11)$$

where the derivatives are evaluated using the background configuration. For Schwarzschild black holes $\frac{\delta^2 S}{\delta g \delta A}$ vanishes and no diagonalization of above action is needed. However, when $Q \neq 0$, the decoupled variables Z_1 and Z_2 are linear combinations of Δg and ΔA .

The spin $s = \frac{3}{2}$ contribution comes purely from the gravitino and obeys eq.(4) with the potential [12,13]

tions have potentials V_s^+ . We will rely on the fact established in [11] that the quasinormal modes for axial and polar perturbations for gravitational waves (and for electromagnetic ones) are identical and therefore we will study here only the mechanism of cancellation in the one-loop approximation of axial perturbation from spin 1, 3/2, 2. However, a more detailed analysis of this point would be desirable.

$$V_{\frac{3}{2}} = G - \frac{dT_1}{dr_*}, \quad G = \frac{\Delta}{r^6}(\lambda r^2 + 2Mr - 2Q^2), \quad (12)$$

$$T_1 = \frac{1}{F - 2Q} \left[\frac{dF}{dr_*} - \lambda \sqrt{\lambda^2 + 1} \right], \quad (13)$$

$$F = \frac{r^6}{\Delta^{1/2}} G, \quad \lambda = (j_{\frac{3}{2}} - \frac{1}{2})(j_{\frac{3}{2}} + \frac{3}{2}), \quad (14)$$

$$j_{\frac{3}{2}} = l + \frac{3}{2}, \quad (l = 0, 1, 2, \dots). \quad (15)$$

The one-loop corrections to the partition function for the non-extreme RN black hole can be calculated using these wave equations. The total answer will be given by the contribution of each field in the multiplet for each $l = 0, 1, 2, \dots$

$$I_{\text{one-loop}} = (e^{iW})_{\text{one-loop}} = \prod_{l=0}^{\infty} \left\{ \frac{(\det \mathcal{D}_{3/2})^2}{\det \mathcal{D}_2 \det \mathcal{D}_1} \right\}. \quad (16)$$

Here, we have doubled the contribution from the axial perturbations to include the polar perturbations. Those have the same quasinormal frequencies and transmission and absorption coefficients (up to a phase) [11] which leads us to expect that they will give the same contributions to the one-loop action as the axial modes.

For black holes which are far from extreme there is no reason to expect any cancellation between different spin fields. The potentials are completely different. In particular, if one would take $Q = 0$ corresponding to the Schwarzschild black hole it is reasonable to expect that the conformal anomaly expression in eq. (1) will be reproduced. This is possible since the Euler number of the Schwarzschild black hole equals 2. It is difficult however to get anything else from the study of the expression for the one-loop corrections for the non-extreme black holes.

Looking into either form of the potential for spin one and spin two waves as given in equations (2) and (6) for non-extreme black holes, as well as on the gravitino potential, it is difficult to see how they could be related in the extreme limit. In this limit at $M = 1$ we get

$$q_1 = 4 + 2j_1 \quad q_2 = 2 - 2j_2 \quad (17)$$

But even at extreme V_1 is different from V_2 for all l :

$$V_1 - V_2 = \frac{2\Delta}{r^5}(2-r)(l+2). \quad (18)$$

However, *numerical calculations* in [10] have shown that the quasinormal frequency trajectories of the photon, gravitino and graviton with increasing charge meet at the same point in the limit of maximal charge $Q = M$. Moreover, in [14] later the numerical calculations were extended to higher *rapidly damped modes* of nearly extreme black holes and it was found that they *spiral into the value of the extreme black hole as the charge increases*. The behavior of the trajectories was found to change dramatically for the charge $M \geq Q > 0.9M$.

To explain this curious behavior of quasinormal modes for the various spin waves in the background of the near extreme RN black holes, the authors of [10] and [15] were able to reorganize the form of the potentials in the wave equations in such a way as to explain their previous results in an analytic way for the extreme black holes with $M = Q = 1$ with horizon at $r = 1$. First of all the potential for all 3 fields is now given in terms of only one function of r

$$f = \frac{r-1}{r^2}. \quad (19)$$

The potentials for the extreme RN solution acquire the form

$$V_1 = +(j_1 + 1) \frac{df}{dr_*} - 4f^3 + (j_1 + 1)^2 f^2, \quad (20)$$

$$V_{\frac{3}{2}} = +(j_{\frac{3}{2}} + \frac{1}{2}) \frac{df}{dr_*} - 4f^3 + (j_{\frac{3}{2}} + \frac{1}{2})^2 f^2, \quad (21)$$

$$V_2 = -j_2 \frac{df}{dr_*} - 4f^3 + j_2^2 f^2, \quad (22)$$

$$j_s = l + s, \quad (l = 0, 1, 2, \dots), \quad (23)$$

and the difference between V_1 and V_2 becomes

$$V_1 - V_2 = \frac{2\Delta}{r^2}(2-r)(l+2) = (j_1 + j_2 + 1) \frac{df}{dr_*}. \quad (24)$$

The tortoise coordinate r_* in terms of which the wave differential operator is very simple for the extreme RN black hole is

$$r_* = r - 1 + \ln(r-1)^2 - \frac{1}{r-1}. \quad (25)$$

Of crucial importance is that r_* maps infinity to infinity and the horizon to negative infinity:

$$r \longrightarrow \infty \quad \Longrightarrow \quad r_* \longrightarrow \infty \quad (26)$$

$$r \longrightarrow 1 \quad \Longrightarrow \quad r_* \longrightarrow -\infty. \quad (27)$$

Also the relation

$$r_* \longrightarrow -r_* \quad \Longrightarrow \quad r - 1 \longrightarrow \frac{1}{r - 1} \quad (28)$$

will prove to be very useful.

Fortunately for supersymmetry, as we will see later, the wave equation for the radial coordinate has a simple second order differential operator in tortoise coordinate as given in \mathcal{D}_s in eq. (4).

These three potentials are related in the following way:

$$V_1(r_*, j_1 = j) = V_{\frac{3}{2}}(r_*, j_{\frac{3}{2}} = j + \frac{1}{2}) = V_2(-r_*, j_2 = j + 1), \quad (29)$$

where j is a positive integer. The first equality is obvious in eqs. (20) and (21); the potential of the Rarita-Schwinger field is identical to that of the spin one field if we shift the multipole index by $1/2$. The second equality is proved in [15] by using the relation $f(r_*) = f(-r_*)$ which follows from (28). Therefore V_2 can be obtained by reflecting V_1 or $V_{\frac{3}{2}}$ about $r_* = 0$. *This transformation corresponds to the exchange of the horizon and infinity.* It has been deduced from eq. (29) in [15] that a scattering problem for each perturbed field with a corresponding multipole index results in the same transmission and reflection amplitudes. This was also derived very recently using the unbroken supersymmetry of the extreme RN in [16].

Where does all this bring us with respect to the one-loop corrections to the entropy of the extreme black holes? If not for the strange fact that the potential for the $s = 2$ perturbations are related to the $s = 1$ perturbations by the inversion of the tortoise coordinate, we would not be able to conclude that there are no corrections. However, the fact that in calculating

one-loop Feynman diagrams in the background of the extreme black hole an exchange of the horizon with infinity is needed is rather unusual. Let us look into this closely. We would like to compare the one-loop contribution for every l for $s = 1$ and $s = 2$ perturbations. The relevant part of the path integral is

$$\int d\Phi_1 d\Phi_2 \exp i \int dr_* (\Phi_1 \mathcal{D}_1 \Phi_1 + \Phi_2 \mathcal{D}_2 \Phi_2) \quad (30)$$

where

$$\mathcal{D}_1 = \left[\frac{d^2}{dr_*^2} + \omega^2 - V_1(r_*) \right], \quad \mathcal{D}_2 = \left[\frac{d^2}{dr_*^2} + \omega^2 - V_2(r_*) \right]. \quad (31)$$

Now we may use the facts that the term $\frac{d^2}{dr_*^2}$ is even in r_* and that $V_1(r_*) = V_2(-r_*)$ for the same value of l . Hence, we conclude that

$$\mathcal{D}_1(r_*) = \mathcal{D}_2(-r_*), \quad (32)$$

i.e. not only the potential but also the total wave equations for spin one and spin two are related by the inversion of the tortoise coordinate and therefore by the exchange of the horizon and infinity. Thus the quadratic part of the relevant action for $s = 1$ is

$$S_1 = \int_{-\infty}^{+\infty} dr_* \Phi_1(r_*) \mathcal{D}_1 \left(\frac{d}{dr_*}, r_* \right) \Phi_1(r_*) \quad (33)$$

and the quadratic part of the relevant action for $s = 2$ is

$$S_2 = \int_{-\infty}^{+\infty} dr_* \Phi_2(r_*) \mathcal{D}_2 \left(\frac{d}{dr_*}, r_* \right) \Phi_2(r_*) = \int_{-\infty}^{+\infty} dr_* \Phi_2(r_*) \mathcal{D}_1 \left(-\frac{d}{dr_*}, -r_* \right) \Phi_2(r_*). \quad (34)$$

Since the integral over r_* extends from $-\infty$ to $+\infty$ we may change the integration $r_* \rightarrow -r_*$ variables and get

$$S_2 = \int_{-\infty}^{+\infty} dr_* \Phi_2(-r_*) \mathcal{D}_1 \left(\frac{d}{dr_*}, r_* \right) \Phi_2(-r_*). \quad (35)$$

The last step is to consider the change of variables in the path integral and to integrate instead of $\Phi_2(r_*)$ over $\Phi_2(-r_*)$

$$\int d\Phi_1(r_*) d\Phi_2(-r_*) \exp i \int dr_* (\Phi_1(r_*) \mathcal{D}_1 \Phi_1(r_*) + \Phi_2(-r_*) \mathcal{D}_1 \Phi_2(-r_*)). \quad (36)$$

Introducing the notation

$$\Phi_2(-r_*) = \tilde{\Phi}_2(r_*) \quad (37)$$

we may rewrite the path integral in the form where the contribution from the gravitons and from the photons is the same for each l .

$$\int d\Phi_1(r_*) d\tilde{\Phi}_2(r_*) \exp i \int dr_* (\Phi_1(r_*) \mathcal{D}_1 \Phi_1(r_*) + \tilde{\Phi}_2(r_*) \mathcal{D}_1 \tilde{\Phi}_2(r_*)) \quad (38)$$

Since the wave equation for the radial mode of the gravitino for each l coincides with the equation for $s = 1$ we conclude that we have now proved the total (axial) cancellation of the one-loop correction to the on-shell action of $N = 2$ supergravity in the extreme RN background. Assuming that the polar modes give the same contributions as the axial ones we find

$$I_{\text{one-loop}} = (e^{iW})_{\text{one-loop}} = \prod_{l=0}^{\infty} \left\{ \frac{(\det \mathcal{D}_{3/2})^2}{\det \mathcal{D}_2 \det \mathcal{D}_1} \right\} = \prod_{l=0}^{\infty} \left\{ \frac{\det \mathcal{D}_{3/2}}{\det \mathcal{D}_1} \right\}^2 = 1. \quad (39)$$

The non-trivial part of this analysis was the use of the coordinate system where the horizon of the extreme black hole is pushed to infinity which allows a particular change of variables in the one-loop path integral.

Thus, the cancellation of the one-loop corrections to the on-shell action (black hole entropy) found here supports the general argument of the supersymmetric non-renormalization theorem [4,2,5]. The theorem states that the extreme black hole entropy does not have corrections in any loop order in supergravity theory. This is in complete agreement with the topological character of the entropy of black holes with unbroken supersymmetry [17].

It would be interesting to see whether a similar cancellation can be established in extremal but non-supersymmetric black holes as suggested in [18]. However, the perturbation equations will be very different and without the hidden help of unbroken supersymmetries it is not clear whether the arguments presented in this paper can be repeated.

The striking similarity between the spin $1, \frac{3}{2}$ and spin 2 perturbations in the extremal supersymmetric case certainly has a deeper explanation in the unbroken supersymmetry of the background. In the present case this was established in [16].

We will now set up a *general framework to relate various perturbations via the unbroken residual supersymmetry*. As the Killing spinor, which is the transformation parameter, is neither constant nor an arbitrary function in supergravity, we are dealing with *rigid supersymmetry*. We are using here the condensed notation introduced by B. De Witt [19] and developed in the context of the background field method for gauge theories in [20]. The underlying action $S(\phi, \psi)$ depending on bosonic and fermionic fields ϕ^i and ψ^a is invariant under the supersymmetry transformations

$$\begin{aligned}\delta\phi^i &= R_\alpha^i(\phi, \psi)\epsilon^\alpha \\ \delta\psi^a &= R_\alpha^a(\phi, \psi)\epsilon^\alpha\end{aligned}\tag{40}$$

where R denote field dependent matrices and ϵ^α are the supersymmetry parameters. If we are interested in bosonic and fermionic perturbations $\Delta\phi^i$ and $\Delta\psi^a$ propagating in a background given by ϕ_0, ψ_0 it is useful to expand the action and the supersymmetry transformations.

For a purely bosonic background with $\psi_0 = 0$ the expansion of the action is given by

$$S(\phi, \psi) = S^{(0)}(\phi_0, \psi_0 = 0) + S^{(2)}(\Delta\phi, \Delta\psi, \phi_0, \psi_0 = 0) + \dots\tag{41}$$

where $S^{(2)}$ is the Gaussian action for the perturbations given by

$$S^{(2)} = \frac{1}{2}\Delta\phi^i S_{,ij}^{(0)} \Delta\phi^j + \frac{1}{2}\Delta\psi^a S_{,ab}^{(0)} \Delta\psi^b.\tag{42}$$

This action eventually gives rise to eqn. (4) and also governs the one-loop corrections. The indices denote derivatives with respect to bosons (i, j, \dots) and fermions (a, b, \dots) . The superscript $^{(0)}$ implies that the relevant expressions are evaluated using the background fields and summation is understood to include integration.

The supersymmetry transformations become

$$\begin{aligned}\delta(\phi^i + \Delta\phi^i) &= R_\alpha^{(0)i}\epsilon^\alpha + R_{\alpha,a}^{(0)i}\Delta\psi^a\epsilon^\alpha \\ \delta(\psi^a + \Delta\psi^a) &= R_\alpha^{(0)a}\epsilon^\alpha + R_{\alpha,i}^{(0)a}\Delta\phi^i\epsilon^\alpha.\end{aligned}\tag{43}$$

$S(\phi, \psi)$ is clearly invariant under the full set of transformations, however we are interested in the symmetries of $S^{(2)}$ in a fixed background. This restricts the transformations to those

which leave the background invariant. The remaining symmetry of $S^{(2)}$ is a rigid supersymmetry where the transformation parameters are the Killing spinors ϵ_K^A of the background. The resulting symmetry transformations on the perturbations are

$$\begin{aligned}\delta\Delta\phi^i &= R_{A,a}^{(0)i}\Delta\psi^a\epsilon_K^A \\ \delta\Delta\psi^a &= R_{A,i}^{(0)a}\Delta\phi^i\epsilon_K^A\end{aligned}\tag{44}$$

where the index A denotes the Killing spinors, rather than the whole set of transformation parameters. These equations relate bosonic and fermionic perturbations or quantum fluctuations. Hence, this residual supersymmetry controls quantum corrections around black holes which admit Killing spinors.

Note that the same formalism applies (in any dimension) to local as well as to global supersymmetries. Then, in the local case the Killing spinors and the residual supersymmetry are rigid, whereas in the global case they are global.

We do not attempt here to make the connection between this general formalism and eqs. (4)-(10) more precise. The advantage of using eqs. (4)-(10) is that they are very simple. The advantage of using eqs. (44) is that they are universal and always *connect solutions for bosonic perturbations with those of fermionic perturbations via Killing spinors*. This explains the cancellation of the one-loop corrections to the entropy via residual supersymmetry.

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